Static polarizability vertex and its applications

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Abstract

Using the Lagrangian that was developed on the corresponding principle between the moving medium electrodynamic and quantum field theory the explicit expression for the static polarizability vertex has been obtained. The applications of this vertex for calculations of real Compton scattering amplitude as well as the imaginary part of doubly virtual Compton scattering amplitude have been demonstrated.

1 Introduction

Nowadays a set of the low energy experiments are performed to investigate the nucleon structure within non-perturbative QCD-region. Some of them allows ones not only to investigate the nucleon elastic properties at low Q^2-region but also to see its internal structure by the different polarizabilities measurement [1].

Performing a such kind of the experiments it will be also important to have some theoretical explanation of the measured quantities. One of the models that can be used for the estimation of Q^2-dependence of the nucleon spin polarizabilities is based on the corresponding principle between the moving medium electrodynamic and quantum field theory. Nowadays the Lagrangian describing this theory was constructed, presented in [2] and for the photon-nucleon tensor interaction has a form

$$\mathcal{L}_{eff}^{pol} = -\frac{i\pi}{M}\left(\bar{\psi}\,\gamma^{\mu}\,\overleftrightarrow{\partial}_{\nu}\,\psi\right)(\alpha_0 F_{\mu\rho}F^{\rho\nu} - \beta_0 \widetilde{F}_{\mu\rho}\widetilde{F}^{\rho\nu}). \tag{1}$$

Here $\overleftrightarrow{\partial}_{\nu} = \overleftarrow{\partial}_{\nu} - \overrightarrow{\partial}_{\nu}$, ψ is the wave functions of the nucleon whose mass is M, $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is electromagnetic tensor, $\widetilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}/2$. Presented above Lagrangian depends on the static electric α_0 and magnetic β_0 polarizabilities whose numerical value are known (see [1, 3] and references therein).

Having Lagrangian that described any interaction between particles we can use the standard Feynman rule technic. To apply presented above Lagrangian for this method in the present report we define the vertex of interaction and consider its application for calculation of real Compton scattering (RCS) and doubly virtual Compton scattering (VVCS).

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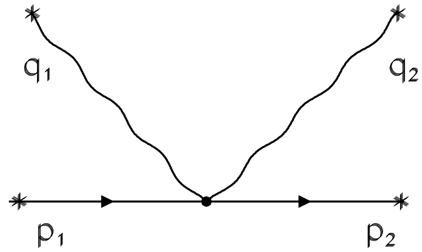

Figure 1: Polarizability vertex

2 Static polarizability vertex

In order to obtain the explicit expression for the polarizability vertex we will follow to the notations of Appendix B of [4]. According to this approach our Lagrangian (1) can be presented as:

$$\mathcal{L}^{pol}_{eff} = \int \prod_{i=1}^{4} \left[d^4 x_i \delta^4(x - x_i) \right] \alpha^{r'r}_{\sigma\delta}(x_1, x_2, x_3, x_4) \; \bar{\psi}_{r'} \; (x_3) \psi_r(x_1) A^{\sigma}(x_4) A^{\delta}(x_2), \tag{2}$$

where $r'r$ are the four-spinor indexes (that usually dropped). Taking into account that

$$\widetilde{F}_{\mu\rho} \widetilde{F}^{\rho\nu} = F_{\mu\rho} F^{\rho\nu} + \frac{1}{2} \delta^{\nu}_{\mu} F_{\rho\sigma} F^{\rho\sigma} \tag{3}$$

we can immediately find that

$$\begin{aligned} \alpha^{r'r}_{\sigma\delta}(x_1, x_2, x_3, x_4) \;\; &= \;\; -\frac{i\pi}{M} \left(\frac{\partial}{\partial x_{3\nu}} - \frac{\partial}{\partial x_{1\nu}} \right) \gamma^{r'r}_{\mu} \left((\alpha_0 - \beta_0) \left[\frac{\partial}{\partial x_{4\mu}} \delta^{\rho}_{\sigma} - \frac{\partial}{\partial x_{4\rho}} \delta^{\mu}_{\sigma} \right] \right. \\ & \quad \times \left[\frac{\partial}{\partial x^{\rho}_{2}} g_{\nu\delta} - \frac{\partial}{\partial x^{\nu}_{2}} g_{\rho\delta} \right] - \frac{1}{2} \delta^{\mu}_{\nu} \beta_0 \left[\frac{\partial}{\partial x_{4\rho}} \delta^{\gamma}_{\sigma} - \frac{\partial}{\partial x_{4\gamma}} \delta^{\rho}_{\sigma} \right] \\ & \quad \left. \times \left[\frac{\partial}{\partial x^{\rho}_{2}} g_{\gamma\delta} - \frac{\partial}{\partial x^{\gamma}_{2}} g_{\rho\delta} \right] \right) \\ &= \;\; \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \tilde{\alpha}^{r'r}_{\sigma\delta}(p_1, q_1, p_2, q_2) \times \\ & \quad \times e^{ip_1(x - x_1) + iq_1(x - x_2) - ip_2(x - x_3) - iq_2(x - x_4)}, \end{aligned} \tag{4}$$

where p_1 and q_1 (p_2 and q_2) are the incoming (outgoing) nucleon and photon momenta respectively (see Fig. 1). As a result $\tilde{\alpha}^{r'r}_{\sigma\delta}(p_1, q_1, p_2, q_2)$ can be found by the following substitution:

$$\begin{aligned} \tilde{\alpha}^{r'r}_{\sigma\delta}(p_1, q_1, p_2, q_2) \;\; &= \;\; \alpha^{r'r}_{\sigma\delta}(x_1, x_2, x_3, x_4) \left[\frac{\partial}{\partial x_1} \to ip_1, \; \frac{\partial}{\partial x_2} \to iq_1, \; \frac{\partial}{\partial x_3} \to -ip_2, \; \frac{\partial}{\partial x_4} \to -iq_2 \right] \\ &= \;\; -\frac{\pi}{M} (p^{\nu}_1 + p^{\nu}_2) \gamma^{r'r}_{\mu} \left((\alpha_0 - \beta_0) [q^{\mu}_2 \delta^{\rho}_{\sigma} - q^{\rho}_2 \delta^{\mu}_{\sigma}] [q_{1\rho} g_{\nu\delta} - q_{1\nu} g_{\rho\delta}] \right. \\ & \quad \left. -\frac{1}{2} \delta^{\mu}_{\nu} \beta_0 \left[q^{\rho}_2 \delta^{\gamma}_{\sigma} - q^{\gamma}_2 \delta^{\rho}_{\sigma} \right] \left[q_{1\rho} g_{\gamma\delta} - q_{1\gamma} g_{\rho\delta} \right] \right). \end{aligned} \tag{5}$$

Summing up over the all symmetric states and multiply the result on i we receive the final expression for polarizability vertex that presented on Fig. 1

$$\begin{aligned} \Gamma^{pol}_{\sigma\delta}(p_1, q_1, p_2, q_2) \;\; &= \;\; i(\tilde{\alpha}_{\sigma\delta}(p_1, q_1, p_2, q_2) + \tilde{\alpha}_{\delta\sigma}(p_1, q_1, p_2, q_2) + \tilde{\alpha}_{\sigma\delta}(p_1, -q_2, p_2, -q_1) + \\ & \quad \tilde{\alpha}_{\delta\sigma}(p_1, -q_2, p_2, -q_1)). \end{aligned} \tag{6}$$

Here we dropped the four-spinor indexes.

3 RCS

As a simplest application for the presented above vertex let us consider RCS

$$\gamma(k) + N(p) \to \gamma(k') + N(p') \tag{7}$$

($k^2 = k'^2 = 0$, $p^2 = p'^2 = M^2$) that in lowest order are described by the following matrix element:

$$\begin{aligned} iT &= \frac{1}{4}\varepsilon^{\mu *}\varepsilon^{\nu}\bar{u}(p')\Gamma^{pol}_{\mu\nu}(p,k,p',k')\bar{u}(p') = -i\varepsilon^{\mu *}\varepsilon^{\nu}\frac{\pi}{M}\bar{u}(p')(2M\beta_0(K_\mu K_\nu - Q_\mu Q_\nu) \\ &+(\alpha_0 - b_0)[\gamma\cdot K(P_\mu K_\nu + P_\nu K_\mu) - 2K^2(\gamma_\mu P_\nu + \gamma_\nu P_\mu) + K\cdot P(\gamma_\mu K_\nu + \gamma_\nu K_\mu)] \\ &-2g_{\mu\nu}[2M\beta_0 K^2 + (\alpha_0 - \beta_0)\gamma\cdot K K\cdot P])u(p), \end{aligned} \tag{8}$$

where we use the standard notations [3] $P = \frac{1}{2}(p+p')$, $K = \frac{1}{2}(k+k')$, $Q = \frac{1}{2}(p-p') = \frac{1}{2}(k'-k)$. Taking into account

$$\bar{u}(p')\gamma\cdot P u(p) = M\bar{u}(p')u(p),\ \bar{u}(p')\gamma\cdot Q u(p) = 0,\ Q^2 = -K^2,\ P\cdot Q = 0 \tag{9}$$

one can see that this result is agree with amplitude obtained in [2].

4 VVCS

Today one of the most interesting topic in polarizability investigation is the measurement of Q^2-dependence of the forward polarizabilities in deep-inelastic scattering (DIS) [5] that can be presented as the imaginary part of VVCS [1]. To shed light on the question about static and dynamic polarizability relations we propose to express the forward polarizabilities via static ones applying the standard Feynman rule technic.

The lowest order VVCS amplitudes with static polarizabilities to whose imaginary part gives non-zero contribution to the hadronic tensor of inclusive DIS are presented by Feynman grafs on Fig. 2. It should be noticed that contribution from Fig. 2 (e) is negligible ($\sim 10^{-8}$ fm^6) and can be dropped. Performing cut over dash line on Fig. 2 (a-d) the imaginary part of these amplitudes can be presented as

$$\mathrm{Im}\, T = \pi^2\varepsilon^*_\nu(q)\varepsilon_\mu(q)\int d\Theta\bar{u}(p)\left[\frac{\Gamma^{\mu\nu\, a}_{VVCS} + \Gamma^{\mu\nu\, b}_{VVCS}}{(p+q)^2 - M^2} + \frac{\Gamma^{\mu\nu\, c}_{VVCS} + \Gamma^{\mu\nu\, d}_{VVCS}}{(p-k)^2 - M^2}\right]u(p), \tag{10}$$

where $d\Theta = d^4k\delta(k^2)\delta((p+q-k)^2 - M^2)$,

$$\begin{aligned} \Gamma^{\mu\nu\, a}_{VVCS} &= \Gamma^{pol\ \nu\alpha}(p+q-k,k,p,q)(\hat{p}+\hat{q}-\hat{k}+M)\Gamma^{el}_\alpha(-k)(\hat{p}+\hat{q}+M)\Gamma^{el\ \mu}(q), \\ \Gamma^{\mu\nu\, b}_{VVCS} &= \Gamma^{el\ \nu}(-q)(\hat{p}+\hat{q}+M)\Gamma^{el}_\alpha(k)(\hat{p}+\hat{q}-\hat{k}+M)\Gamma^{pol\ \mu\alpha}(p,q,p+q-k,k), \\ \Gamma^{\mu\nu\, c}_{VVCS} &= \Gamma^{pol\ \nu\alpha}(p+q-k,k,p,q)(\hat{p}+\hat{q}-\hat{k}+M)\Gamma^{el\ \mu}(q)(\hat{p}-\hat{k}+M)\Gamma^{el}_\alpha(-k), \\ \Gamma^{\mu\nu\, d}_{VVCS} &= \Gamma^{el}_\alpha(k)(\hat{p}-\hat{k}+M)\Gamma^{el\ \nu}(-q)(\hat{p}+\hat{q}-\hat{k}+M)\Gamma^{pol\ \mu\alpha}(p,q,p+q-k,k), \end{aligned} \tag{11}$$

and $\Gamma^{el}_\mu(q) = -ie\,(F_D(-q^2)\gamma_\mu + F_P(-q^2)i\sigma_{\mu\alpha}q^\alpha/2M)$ is the usual elastic vertex.

Notice that according to [1] from the expression (10) one can extract the partial cross sections $K\sigma_T$, $K\sigma_L$, $K\sigma_{TT}$, $K\sigma_{LT}$ and calculate Q^2-dependences of forward polarizabilities in

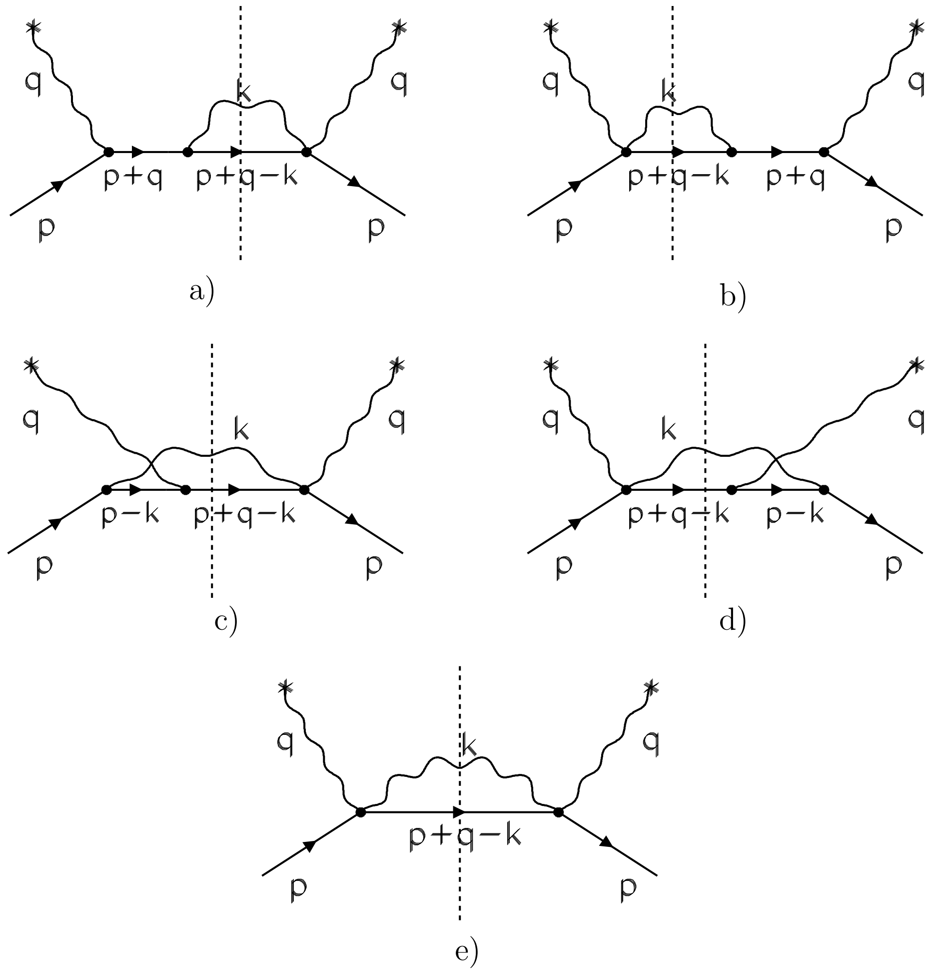

Figure 2: Feynman graphs for the lowest order VVCS amplitudes whose imaginary parts give contributions to the hadronic tensor for inclusive DIS. The dashed lines show the cuts for imaginary part calculations.

a following way:

$$\alpha(Q^2)+\beta(Q^2)=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty}\frac{K(Q^2,\nu)}{\nu}\frac{\sigma_T(Q^2,\nu)}{\nu^2}d\nu,\ \ \alpha_L(Q^2)=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty}\frac{K(Q^2,\nu)}{\nu}\frac{\sigma_L(Q^2,\nu)}{\nu^2}d\nu,$$

$$\gamma_0(Q^2)=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty}\frac{K(Q^2,\nu)}{\nu}\frac{\sigma_{TT}(Q^2,\nu)}{\nu^3}d\nu,\ \ \delta_{LT}(Q^2)=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty}\frac{K(Q^2,\nu)}{\nu}\frac{\sigma_{LT}(Q^2,\nu)}{Q\nu^2}d\nu. \quad (12)$$

In these expressions $Q^2=-q^2$, $\nu=E-E'$ and E (E') is an incoming (outgoing) electron energy in lab. frame in DIS.

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